Algebraic Geometry FINAL - Back Paper

There are 5 questions of 10 points each. Please look over the entire paper before attempting to answer as some questions may be easier than others.

1. Show that an affine algebraic variety is the intersection of finitely many hypersurfaces.
2. Prove that the complement of a point in $\mathbb{A}^n$ is an open set that is compact in the Zariski topology.
3. Compute the Hilbert Polynomial of $\mathbb{P}^2$.
4. Let R be a commutative ring. Define the Zariski topology on $Spec(R)$ and show that a point is closed if and only if it is a maximal ideal.
5. Find examples of

   i. Two plane curves which are isomorphic as quasi-projective varieties, but have different degrees.

   ii. Two plane curves which have the same degree but are not isomorphic.